UNIVERSITEIT VAN PRETORIA / *UNIVERSITY OF PRETORIA*
DEPT SIVIELE INGENIEURSWESE / *DEPT OF CIVIL ENGINEERING*

STRENGTH OF MATERIALS SWK210 STERKTELEER SWK210
SEMESTER TEST 1 – SEMESTERTOETS 1

VAN en VOORLETTERS	HANDTEKENING	STUDENTENOMMER							
	Memorandum	1	2	3	4	5	6	7	8
SURNAME and INITIALS	SIGNATURE	STUDENT NUMBER							

STUDY DISCIPLINE STUDIERIGTING		TUITION LANGUAGE ONDERRIGTAAL	ENG or / of AFR

Volpunte / *Full Marks:* 60 **Tyd / *Time*:** 1½ ure / *hours* **5 March 2010**

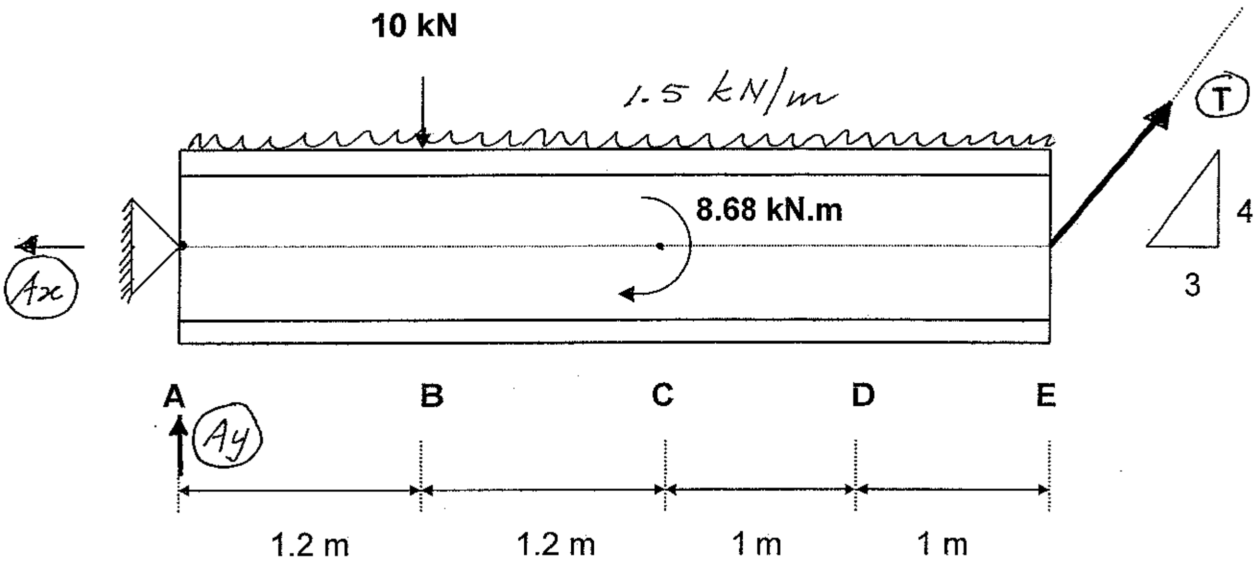
1 **14**	2 **8**	3 **8**	4 **20**		Σ **50**

INSTRUCTIONS ...... READ:
⇨ Answer all questions in the provided spaces.
⇨ The invigilators will supply no additional or loose pages.
⇨ Answers in pencil will not be marked.
⇨ Tippex or any other similar product may not be used.
⇨ No highlighter may be used.
⇨ Students may ask no questions for whatever reason during the exam or test. If you are of the opinion that you need additional information, make assumptions and state these assumptions.
⇨ The relevant units must substantiate all answers.
⇨ All aspects as described in the EXAMINATION REGULATIONS are applicable.
⇨ All calculations to reach an answer must be shown.

INSTRUKSIES....... LEES:
⇨ *Beantwoord alle vrae in die spasies voorsien.*
⇨ *Die toesighouers sal geen addisionele of los bladsye voorsien nie.*
⇨ *Antwoorde in potlood word nie gemerk nie.*
⇨ *Tippex of enige soortgelyke produk mag nie gebruik word nie.*
⇨ *Geen glimpen ["highlighter"] mag gebruik word nie.*
⇨ *Studente mag nie tydens die eksamen vrae vra nie – om watter rede ookal. Indien u van mening is dat addisionele inligting benodig word, maak aannames en stel die aannames.*
⇨ *Alle antwoorde moet deur die nodige eenhede bevestig word.*
⇨ *Alle aspekte soos vervat in die EKSAMENREGULASIES is van toepassing.*
⇨ *Alle berekeninge om antwoorde te bepaal moet getoon word.*

Dosente / Lecturers: Prof BWJ van Rensburg Mr F van Graan Prof L Maree	
Eksterne Eksaminator / External Examiner: Prof WMG Burdzik	

QUESTION 1 / *VRAAG 1* **[14]**



The figure shows an I beam ABCDE that weighs 1.5 kN per metre and which is supported by a hinge at A and a cable at E. A 10 kN vertical load acts at B as well as a couple of magnitude 8.68 kN.m at C.

Die figuur toon 'n I balk ABCDE wat 1.5 kN per meter weeg en wat deur 'n skamier by A en 'n kabel by E ondersteun word. 'n 10 kN vertikale las werk in by B asook 'n koppel van grootte 8.68 kN.m by C.

1[a] Determine all the reactions. **[3]**
Bepaal al die reaksies.

$\odot\ \sum M_A = 0 : -10(1.2) - 1.5(4.4)(2.2) - 8.68 + \frac{T}{5}(4) \times 4.4 = 0$

$\therefore T = 10\,kN \rightarrow$

$\sum M_E = 0 :$

$-4.4\,A_y + 10(3.2) + 1.5(4.4)(2.2) - 8.68 = 0$

$\therefore A_y = 8.6\,kN \rightarrow$

$\odot$ Check:

$\sum F\downarrow = 10 + 1.5(4.4) = 16.6\,kN \quad \sum F\uparrow = 16.6\,kN$

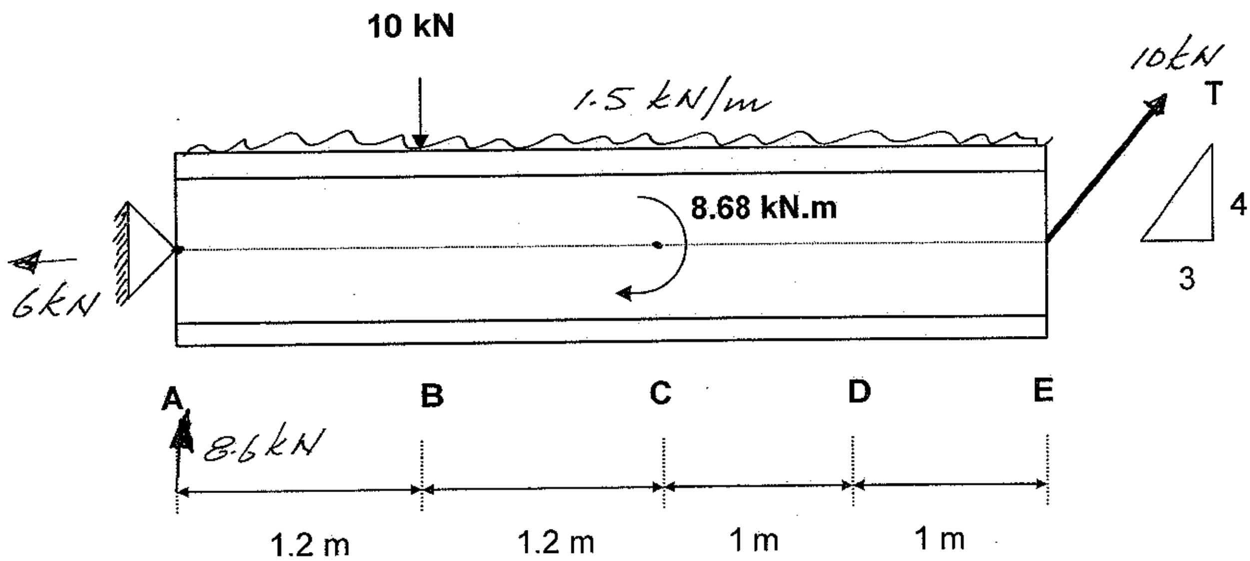
$\odot\ \sum F_x = 0 \quad -A_x + \frac{10}{5}(3) = 0$

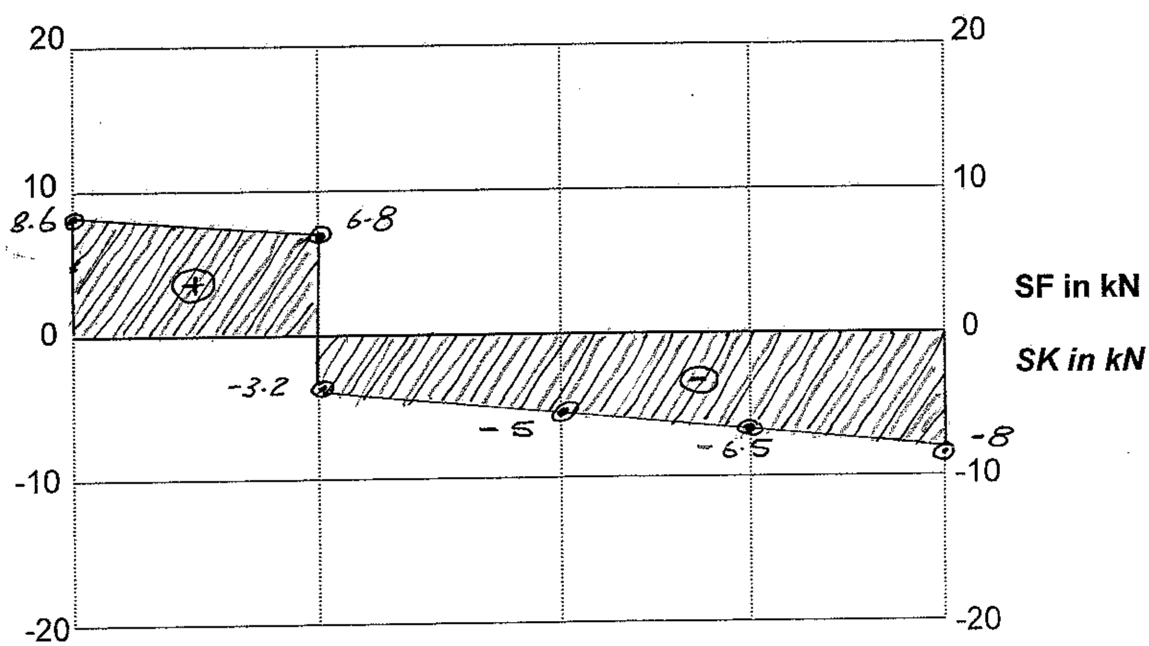
$\therefore A_x = 6\,kN \rightarrow$

1[b] Draw the Shear Force diagram, note all extreme values on the given table and also indicate these values on your Shear Force diagram. **[3]**

Teken die Skuifkragdiagram, noteer alle ekstreemwaardes op die gegewe table en dui ook hierdie waardes aan op u Skuifkragdiagram.

POSITION *POSISIE*	**A**	**B**	**C**	**D**	**E**
SHEAR FORCE / ***SKUIFKRAG IN kN***	0 8.6	6.8 -3.2	-5	-6.5	0 -8

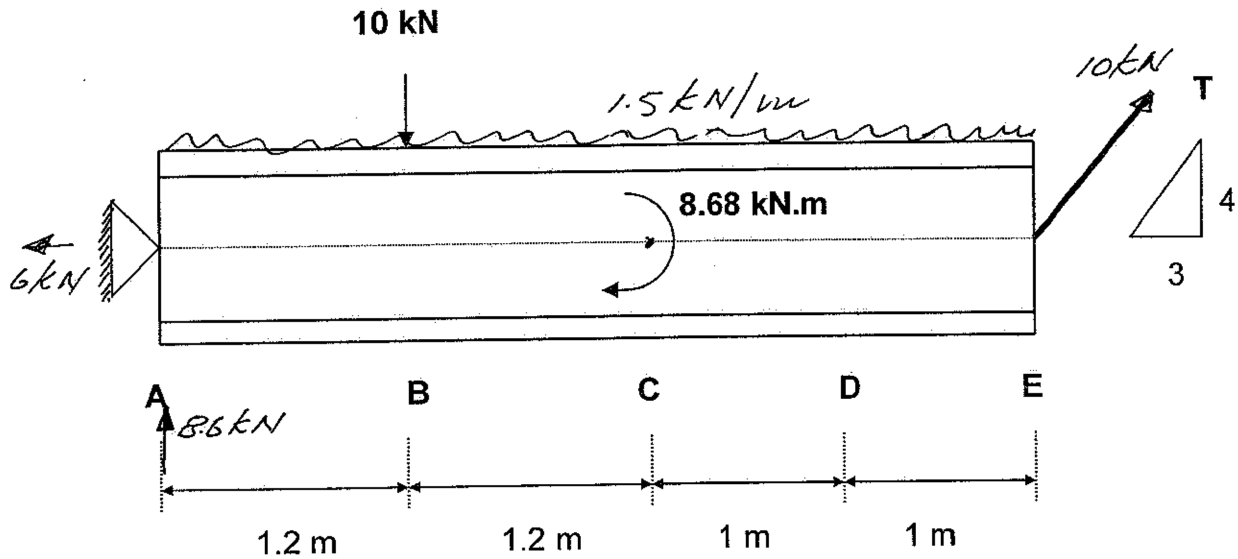


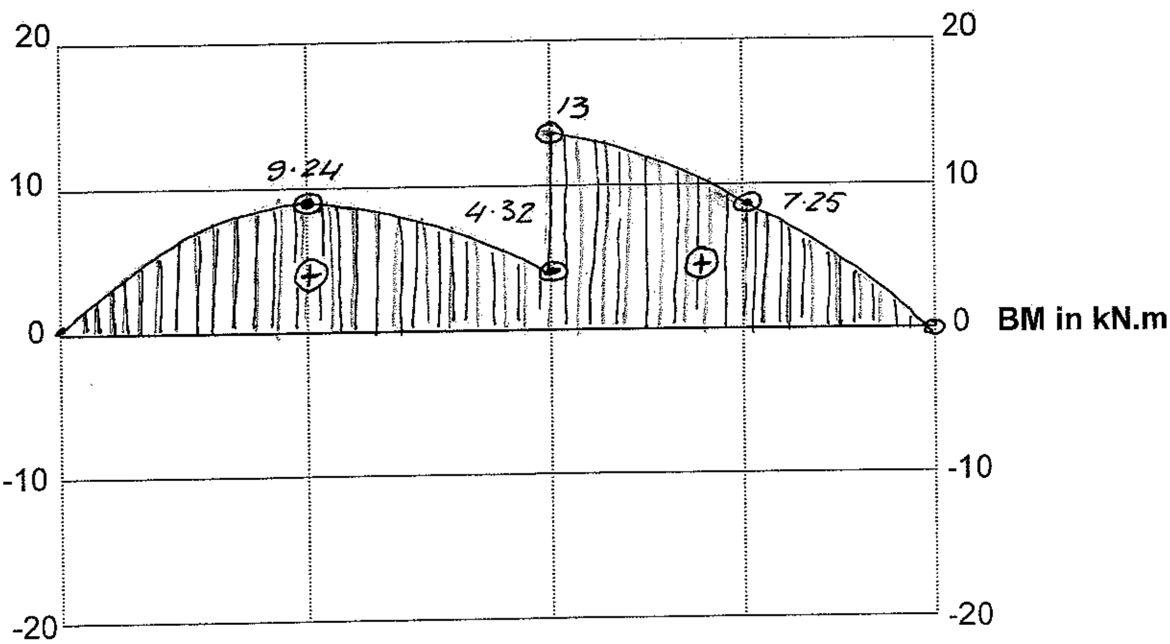


1[c] Draw the Bending Moment diagram, note all extreme values on the given table and also indicate these values on your Bending Moment diagram. **[5]**

Teken die Buigmomentdiagram, noteer alle ekstreemwaardes op die gegewe tabel en dui ook hierdie waardes aan op u Buigmomentdiagram.

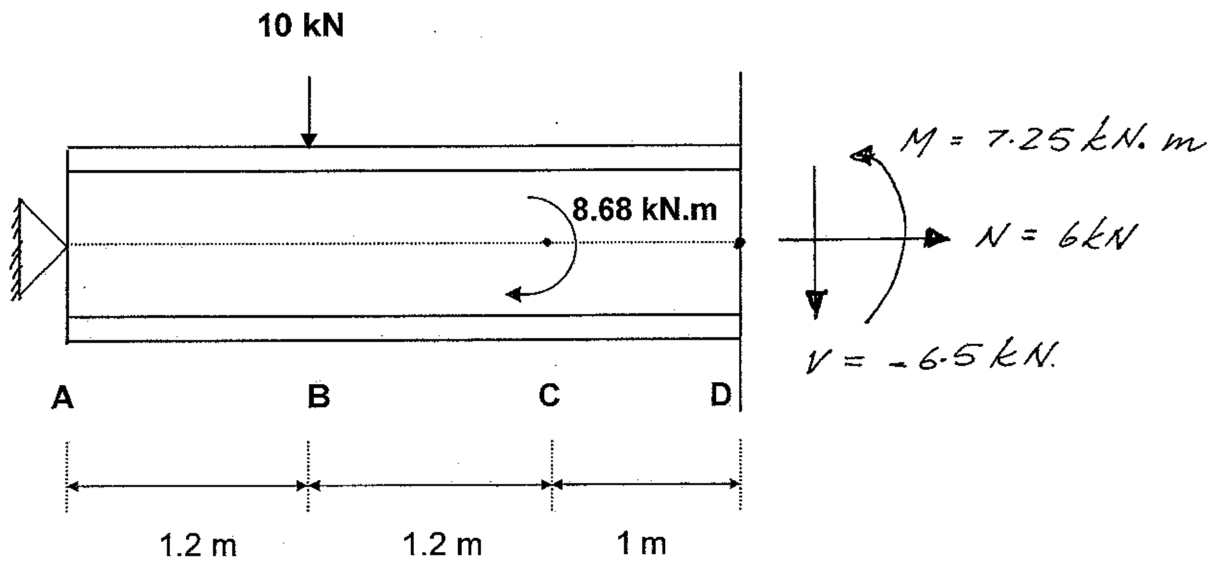
POSITION *POSISIE*	A	B	C	D	E
BENDING MOMENT / ***BUIGMOMENT*** ***IN kN.m.***	0	9·24	13 4·32	7·25	0





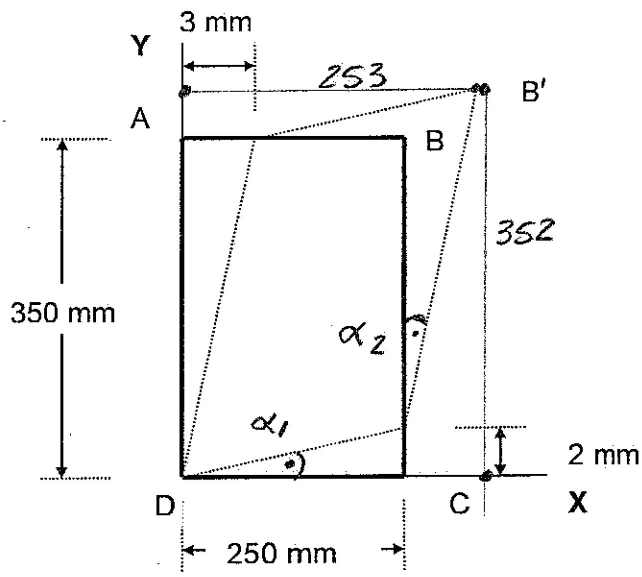
1[d] Determine the resultant internal loadings acting at the cross section through point D on the centre line of the beam. Indicate these loadings as well as their magnitude on the given figure. **[3]**

Bepaal die resultante interne belastings wat op die dwarsdeursnit deur punt D op die hartlyn van die balk inwerk. Dui hierdie belastings asook hulle grootte op die gegewe figuur aan.



QUESTION 2 / *VRAAG 2* **[8]**

The figure shows a rectangular flat metal sheet ABCD [350 mm x 250 mm] and the dashed lines indicate the position after deformation.
Die figuur toon 'n reghoekige plat metaalplaat ABCD [350 mm x 250 mm] en die stippellyne dui die posisie na vervorming aan.



2[a] Calculate the linear strain along diagonal DB. **[5]**
Bereken die lineêre vervorming langs diagonaal DB.

$DB^2 = 350^2 + 250^2 \quad \therefore DB = 430 \cdot 116 \text{ mm} \rightarrow$

$(DB')^2 = 352^2 + 253^2 \quad \therefore DB' = 433 \cdot 489$

$\varepsilon = \dfrac{\Delta L}{L_o} = \dfrac{433 \cdot 489 - 430 \cdot 116}{430 \cdot 116}$

$\therefore \varepsilon = 0 \cdot 007\,842 \rightarrow$

2[b] Calculate the shear strain at point C with respect to the x and y axes. **[3]**
Bereken die skuifvervorming by punt C met betrekking tot die x- en y-asse.

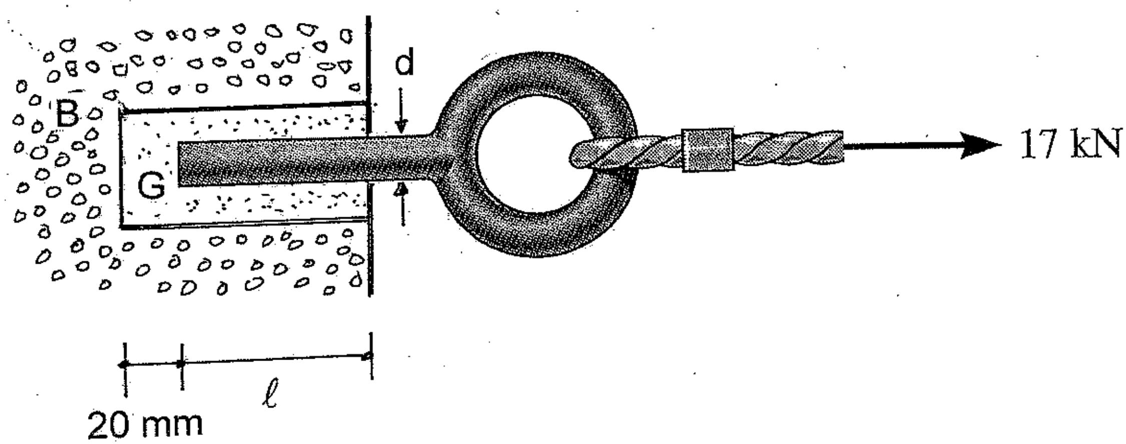
$\gamma_{xy} = \alpha_1 + \alpha_2 = -\left(\frac{3}{350} + \frac{2}{250}\right)$

$= -0 \cdot 016\,571$

QUESTION 3 / *VRAAG 3* **[8]**

The steel anchor bolt with a circular cross section supports a steel cable with a cable force of 17 kN as shown. The anchor bolt is fixed by means of grout [**G**] into the in-situ material [**B**]. The allowable normal stress for the anchor bolt is 160 MPa and the allowable shear stress for the grout is 3 MPa.

Die staal ankerbout met 'n ronde snit ondersteun 'n staal kabel met 'n 17 kN kabelkrag soos getoon. Die ankerbout is met behulp van breivulling [**G**] *in die in-situ materiaal* [**B**] *bevestig.*
Die toelaatbare normaalspanning vir die ankerbout is 160 MPa en die toelaatbare skuifspanning vir die breivulling is 3 MPa.



3[a] Calculate the diameter **[d]** for the bolt to the nearest millimetre. **[3]**
*Bereken die diameter **[d]** vir die bout tot die naaste millimeter.*

$$\sigma = \frac{F}{A}$$

$$\therefore 160 = \frac{17000 \times 4}{\pi d^2}$$

$$\therefore d = 11{\cdot}63 \text{ mm}$$

$$\therefore d = 12 \text{ mm}$$

3[b] Assume that the diameter **[d]** for the bolt is equal to 16 mm: **[3]**
Calculate the minimum length **[ℓ]** to the nearest millimetre that the anchor bolt should be embedded in the in-situ material.
Aanvaar dat die diameter **[d]** *van die die bout gelyk is aan 16 mm:*
Bereken die minimum lengte [ℓ] tot die naaste millimeter wat die ankerbout in die in-situ materiaal bevestig moet word.

$$\tau = \frac{V}{A}$$

$$\therefore 3 = \frac{17000}{\pi * 16 * \ell} \qquad \ell = 112.73$$

$$\therefore \ell = 113 \text{ mm} \longrightarrow$$

3[c] Assume that the diameter **[d]** for the bolt is equal to 16 mm, that the diameter of the hole is 25 mm and that the length **[ℓ]** that the anchor bolt should be embedded in the in-situ material is equal to 140 mm: **[2]**
Calculate the volume grout [in mm^3] necessary to fill the hole.

Aanvaar dat die diameter **[d]** *van die die bout gelyk is aan 16 mm, dat die diameter van die gat gelyk is aan 25 mm en dat die lengte [ℓ] wat die ankerbout in die in-situ materiaal bevestig moet word gelyk is aan 140 mm:*
Bereken die volume breivulling [in mm^3] nodig om die gat te vul.

$$V = \frac{\pi}{4}(25)^2 * 160 - \frac{\pi}{4}(16)^2 * 140$$

$$= 50\,392 \text{ mm}^3 \longrightarrow$$

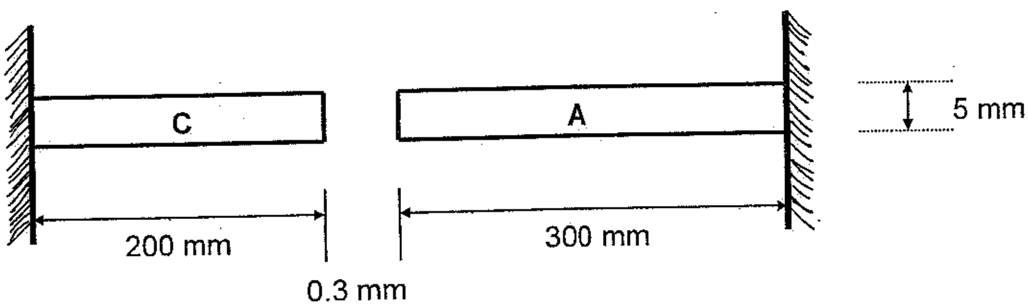
QUESTION 4 / *VRAAG 4* **[20]**

A rectangular aluminium rod **[A]** [300 x 15 x 5] mm and a rectangular copper rod **[C]** [200 x 15 x 5] mm are fixed in-line to two rigid walls as shown. The gap between the rods is 0.3 mm. Room temperature is 22 °C.

*'n Reghoekige aluminium stang **[A]** [300 x 15 x 5] mm en 'n reghoekige koper stang **[C]** [200 x 15 x 5] mm is in lyn geheg aan twee star mure soos getoon. Die gaping tussen die stawe is 0.3 mm. Kamertemperatuur is 22 °C.*

$\alpha_A = 24 \times 10^{-6} / °C$ $\alpha_C = 17 \times 10^{-6} / °C$

$E_A = 70$ GPa $E_C = 125$ GPa



4[a] Calculate the temperature necessary to just close the 0.3 mm gap. **[4]**
Bereken die temperatuur nodig om die 0.3 mm gaping net te sluit.

$\Delta L_{A_T} + \Delta L_{C_T} = 0.3 \text{ mm}$

$\therefore 300 * 24 * 10^{-6} * \Delta T + 200 * 17 * 10^{-6} * \Delta T = 0.3$

$\therefore 0.0106 \, \Delta T = 0.3$

$\therefore \Delta T = 28.3 °C$

$\therefore T_2 = 50.3 °C \longrightarrow$

4[b] The temperature now rises to 70 °C: Calculate the stress in the aluminium. **[10]**
Die temperatuur styg nou tot 70 °C: Bereken die spanning in die aluminium.

$$(\Delta L_T)_A - (\Delta L_F)_A + (\Delta L_T)_C - (\Delta L_F)_C = 0.3$$

$$\therefore 300(24*10^{-6})48 - \frac{F*300}{15(5)*70*10^3}$$

$$+ 200(17*10^{-6})48 - \frac{F*200}{15(5)*125*10^3} = 0.3$$

$$\therefore 0.000\,078\,476\,F = 0.2088 \quad \therefore F = 2660.69\,N$$

$$\sigma_A = \frac{2660.69}{15\times5} = 35.476\ MPa$$

4[c] What is the length of the aluminium rod now at 70 °C? **[6]**
Wat is die lengte van die aluminium staaf nou by 70 °C?

$$L_A = 300 + 300(24*10^{-6})*48 - \frac{2660.69*300}{15(5)*70*10^3}$$

$$= 300 + 0.3456 - 0.1520$$

$$= 300.1936\ mm$$

Check: L_c

$$L_c = 200 + 200(17*10^{-6})48 - \frac{2660.69*200}{15(5)*125*10^3}$$

$$= 200 + 0.1632 - 0.0568 = 200.1064\ mm$$

$$L_A + L_c = 300.1936 + 200.1064$$

$$= 500.30\ mm$$ Correct.

Hooke's Law axial stress

$$\sigma = E\varepsilon$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

Thermal expansion

$$\varepsilon_T = \alpha(\Delta T)$$

Strain Energy (Axial)

$$U = \frac{P^2 L}{2EA} = \frac{EA\delta^2}{2L}$$

Strain Energy Density (Axial)

$$u = \frac{\sigma^2}{2E} = \frac{E\varepsilon^2}{2}$$

Impacct Loading

$$\delta_{\max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{\frac{1}{2}}\right]$$

$$\delta_{st} = \frac{WL}{EA}$$

$$\sigma_{\max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{\frac{1}{2}}\right]$$

$$\sigma_{st} = \frac{W}{A}$$

Torsion Formulas

$$\tau_{\max} = \frac{Tr}{I_P}$$

$$\theta = \frac{T}{GI_P}$$

$$\phi = \frac{TL}{GI_P}$$

Power Transmission

$$P = 2\pi f T \quad (f \; in \; Hz)$$

Strain Energy (Torsional)

$$U = \frac{T^2 L}{2GI_P} = \frac{GI_P\phi^2}{2L}$$

Strain Energy Density (Torsional)

$$u = \frac{\tau^2}{2G} = \frac{G\gamma^2}{2}$$

Flexure Formula

$$\sigma_x = -\frac{My}{I}$$

Section moduli

$$S_i = \frac{I}{c_i} \qquad \text{(i=1,2)}$$

Shear Formula (Rectangular beam)

$$\tau = \frac{VQ}{Ib}$$

Distribution of shear stress (Rectangular beam)

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

Shear Flow

$$f = \frac{VQ}{I}$$

Composite Beams

$$\sigma_{xi} = \frac{MyE_i}{E_1 I_1 + E_2 I_2}$$

(i=1,2)

Mohr's Circle:
Center at

$$\frac{\sigma_x + \sigma_y}{2}$$

Radius

$$R = \left(\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}{}^2\right)^{\frac{1}{2}}$$

Hooke's Law for Plane Stress:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right)$$

Plane stress energy densit

$$u = \tfrac{1}{2}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z\right)$$

Dilatation:

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Pressure Vessels
Spherical:

$$\sigma = \frac{\Pr}{2t}$$

$$\tau_{\max} = \frac{\sigma}{2}$$

Cylindrical:

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\tau_{\max} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

Deflection of Beams

$$EI\frac{d^2 v}{dx^2} = M(x)$$