UNIVERSITEIT VAN PRETORIA / *UNIVERSITY OF PRETORIA*
DEPT SIVIELE INGENIEURSWESE / *DEPT OF CIVIL ENGINEERING*

STRENGTH OF MATERIALS SWK210 STERKTELEER SWK210
SEMESTER TEST 1 – SEMESTERTOETS 1

VAN en VOORLETTERS	HANDTEKENING	STUDENTENOMMER							
	Memorandum	1	2	3	4	5	6	7	8
SURNAME and INITIALS	SIGNATURE	STUDENT NUMBER							

Volpunte / *Full Marks:* 60 Tyd / *Time*: 1½ ure / *hours* 6 March 2009

1	2	3	4	5	Σ
17	13	8	12	10	60

INSTRUCTIONS ...... READ:

⇨ Answer all questions in the provided spaces.
⇨ The invigilators will supply no additional or loose pages.
⇨ Rough work may be done on the final blank page but this page will not be marked.
⇨ Answers in pencil will not be marked.
⇨ Tippex or any other similar product may not be used.
⇨ No highlighter may be used.
⇨ Students may ask no questions for whatever reason during the exam or test. If you are of the opinion that you need additional information, make assumptions and state these assumptions.
⇨ The relevant units must substantiate all answers.
⇨ All aspects as described in the EXAMINATION REGULATIONS are applicable.
⇨ All calculations to reach an answer must be shown.

INSTRUKSIES....... LEES:

⇨ *Beantwoord alle vrae in die spasies voorsien.*
⇨ *Die toesighouers sal geen addisionele of los bladsye voorsien nie.*
⇨ *Rofwerk mag op die laaste blanko bladsy gedoen word en hierdie bladsy word nie gemerk nie.*
⇨ *Antwoorde in potlood word nie gemerk nie.*
⇨ *Tippex of enige soortgelyke produk mag nie gebruik word nie.*
⇨ *Geen glimpen ["highlighter"] mag gebruik word nie.*
⇨ *Studente mag nie tydens die eksamen vrae vra nie – om watter rede ookal. Indien u van mening is dat addisionele inligting benodig word, maak aannames en stel die aannames.*
⇨ *Alle antwoorde moet deur die nodige eenhede bevestig word.*
⇨ *Alle aspekte soos vervat in die EKSAMENREGULASIES is van toepassing.*
⇨ *Alle berekeninge om antwoorde te bepaal moet getoon word.*

Dosente / Lecturers: Mr F van Graan	Prof C Roth	Prof L Maree
Eksterne Eksaminator / External Examiner: Prof BWJ VAN RENSBURG		

QUESTION 1 / VRAAG 1

[17]

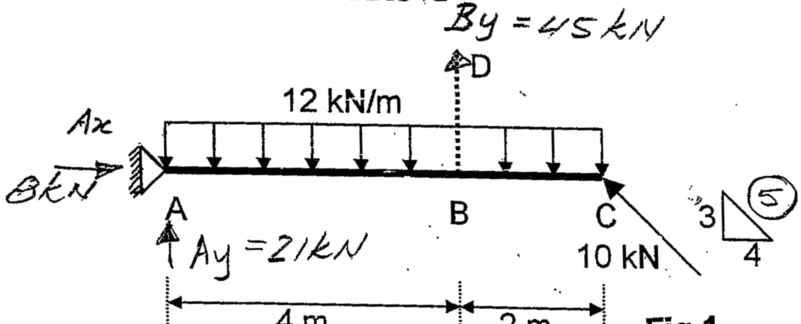


Fig 1

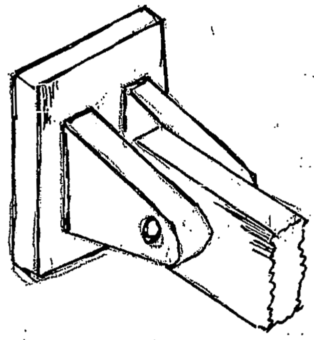


Fig 2

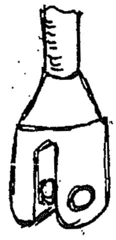


Fig 3

Beam ABC is supported by a hinge at A and a cable BD at B.
Balk ABC word ondersteun deur 'n skarnier by A en 'n kabel BD by B.

1[a] Calculate the reactions at A and B. **[3]**
Bereken die reaksies by A en B.

$\sum M_A = 0: \quad 4B_y - 6(12)(3) + \frac{10}{5}(3)(6) = 0 \qquad \therefore B_y = 45\text{kN}$

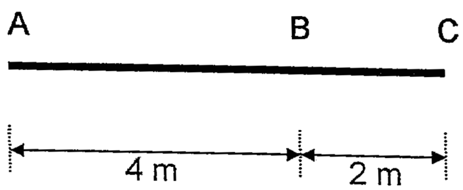
$\sum M_B = 0: \quad -4A_y + 12(6)(1) + \frac{10}{5}(3)(2) = 0 \qquad \therefore A_y = 21\text{ kN}$

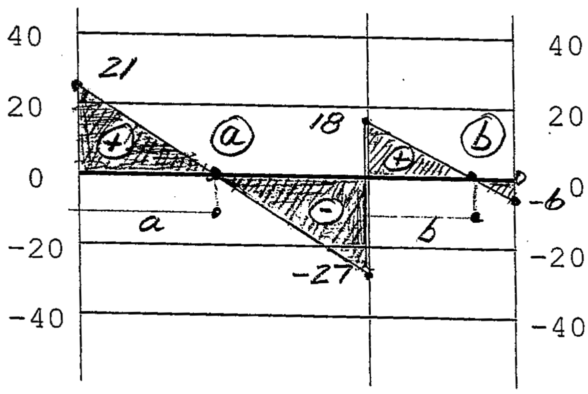
Test: $\sum F_y \downarrow = 6(12) = 72\text{ kN} \qquad \sum F_y \uparrow = 45 + 21 + 6 = 72\text{kN}$

$\sum F_x = 0 \qquad \therefore A_x = \frac{10}{5}(4) = 8\text{kN} \rightarrow$

1[b] Draw the Shear Force and Bending Moment Diagram for beam ABC. Give all extreme values. **[8]**

Teken die Skuifkrag- en Buigmomentdiagram vir balk ABC. Gee alle ekstreemwaardes.





Shear Force in kN
Skuifkrag in kN

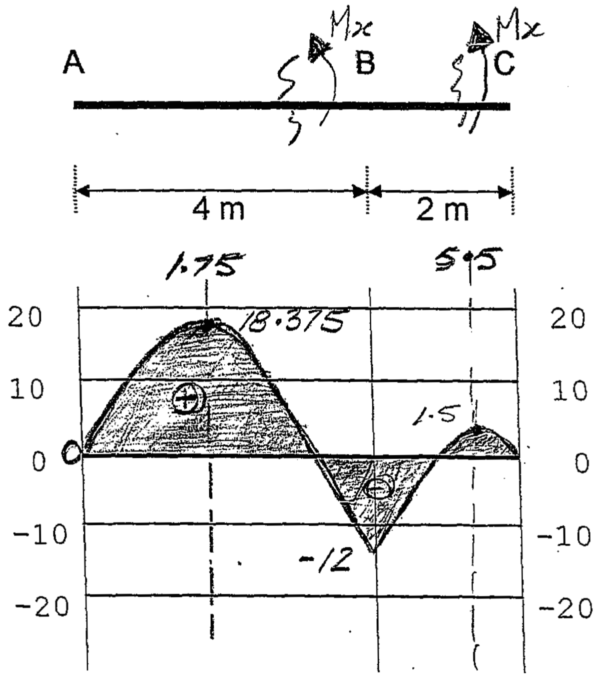
ⓐ: $\frac{21}{a} = \frac{27}{4-a}$

$\therefore 84 - 21a = 27a \qquad a = 1.75\text{ m}$

ⓑ: $\frac{18}{b} = \frac{6}{2-b}$

$36 - 18b = 6b$

$b = 1.5\text{ m}$



Bending Moment in kN.m

Buigmoment in kN.m

(AB) $: -21x + \frac{12x^2}{2} + M_x = 0$

$M_x = 21x - 6x^2$

$x = 1.75\,m : \quad M_x = 18.375\,kN.m$

$x = 4\,m : \quad M_x = -12\,kN.m$

(BC) $: -21x + \frac{12x^2}{2} - 45(x-4) + M_x = 0$

$M_x = -6x^2 + 66x - 180$

$x = 5.5 \quad M_x = +1.5\,kN.m$

1[c] The hinge at A is depicted by Figure 2. If the allowable shear stress in the steel pin is 90 MPa and the allowable normal stress in the steel pin is 150 MPa, calculate the minimum diameter of the pin. **[3]**

Die skarnier by A word weergegee deur Figuur 2.
Indien die toelaatbare skuifspanning in die pen 90 MPa is en die toelaatbare normaalspanning in die pen 150 MPa is, bereken die minimum diameter van die pen.

$\tau = \frac{V}{A} : \quad V = (8^2 + 21^2)^{\frac{1}{2}} = 22.47\,kN.$

$\therefore 90 = \frac{22.47 \times 10^3}{2(\frac{\pi}{4})\,d^2}$

$d = 12.6\,mm$

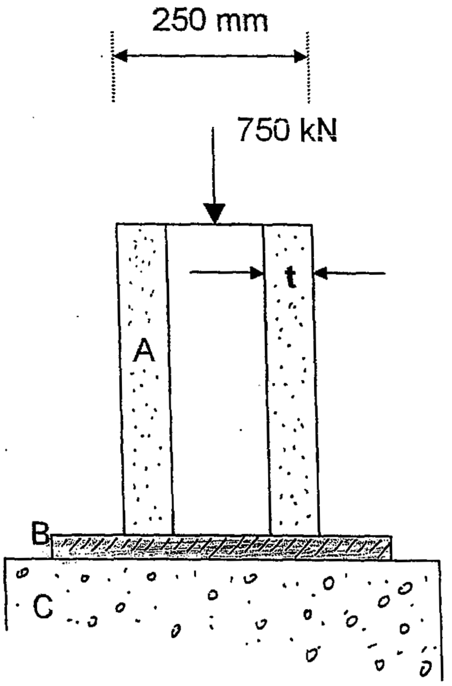
1[d] The cable at B is depicted by Figure 3. If the allowable shear stress in the cable is 90 MPa and the allowable normal stress in the cable is 150 MPa, calculate the minimum diameter of the cable. **[3]**

Die kabel by B word weergegee deur Figuur 3.
Indien die toelaatbare skuifspanning in die kabel 90 MPa is en die toelaatbare normaalspanning in die kabel 150 MPa is, bereken die minimum diameter van die kabel.

$\sigma = \frac{F}{A} : \quad 150 = \frac{45 \times 10^3}{\frac{\pi}{4}(d^2)}$

$\therefore d = 19.54\,mm$

QUESTION 2 / VRAAG 2 **[13]**



$\odot$ Area pipe $= \pi[125^2 - (125-t)^2]$

$\therefore A = \pi(125)^2 - \pi(125)^2 + 2\pi(125t) - \pi t^2$

$= 250\pi t - \pi t^2$

The figure shows a 3.2 metres long hollow, circular steel pipe column [A], outside diameter 250 mm that is supported by a thin circular steel base plate [B] and a reinforced concrete foundation [C].

Die figuur toon 'n 3.2 meter lang hol, ronde staalpyp kolom [A] met 'n buitediameter van 250 mm wat ondersteun word deur 'n dun ronde staal basisplaat [B] en 'n gewapende betonfondament [C].

2[a] If the allowable stress in the steel column is 55 MPa, calculate the minimum wall thickness [**t**] of the pipe. **[6]**

Indien die toelaatbare spanning in die staalkolom 55 MPa is, bereken die minimum wanddikte [**t**] van die pyp.

$\odot \quad \sigma = P/A$

$\therefore 55 = \dfrac{750\ 000}{250\pi t - \pi t^2}$

$\therefore 173t^2 - 43197t + 750\ 000 = 0$

$\therefore t = \dfrac{43197 \pm \sqrt{43197^2 - 4(173)(750\ 000)}}{2(173)}$

$t = \dfrac{43197 \pm 36701}{346} = 18.8 \text{ mm}$

2[b] Assume that the minimum wall thickness [**t**] of the column is 20 mm. **[3]**
Assume that the load causes a uniform stress in the concrete foundation and that the allowable bearing stress of the concrete foundation is 11.5 MPa.
Calculate the minimum diameter [D] of the circular steel base plate [B].

Aanvaar dat die minimum wanddikte [t] van die kolom 20 mm is. Aanvaar dat die las 'n uniforme druk op die betonfondament uitoefen en dat die toelaatbare draspanning van die betonfondament 11.5 MPa is. Bereken die minimum diameter [D] van die ronde staal basisplaat [B].

$\sigma = \frac{F}{A} \quad \therefore 11.5 = \frac{750 * 10^3}{\frac{\pi}{4} d^2}$

$\therefore d = 288 \text{ mm} \rightarrow$

2[c] Assume that the minimum wall thickness of the column is 20 mm, that Young's Modulus for steel [E] is 210 GPa and that Poisson's Ratio [ν] for steel is 0.35. **[4]**
Calculate the increase in the wall thickness of the column in **μm** as a result of the 750 kN load.

Aanvaar dat die minimum wanddikte van die kolom 20 mm is, Young se Modulus [E] vir staal 210 GPa is en dat Poisson se Verhouding [ν] vir staal 0.35 is. Bereken die toename in wanddikte in **μm** as gevolg van die 750 kN belasting.

$\varepsilon = \frac{\sigma}{E} = \frac{P}{AE} = \frac{750\ 000}{\pi(125^2 - 105^2) * 210 * 10^3}$

$= 0.000\ 247 \rightarrow$

$\varepsilon' = \nu\varepsilon = 0.000\ 247 * 0.35$

$= 0.000\ 086 \rightarrow$

$\therefore \Delta d_{out} = 250 * 0.000\ 086 \quad - ①$

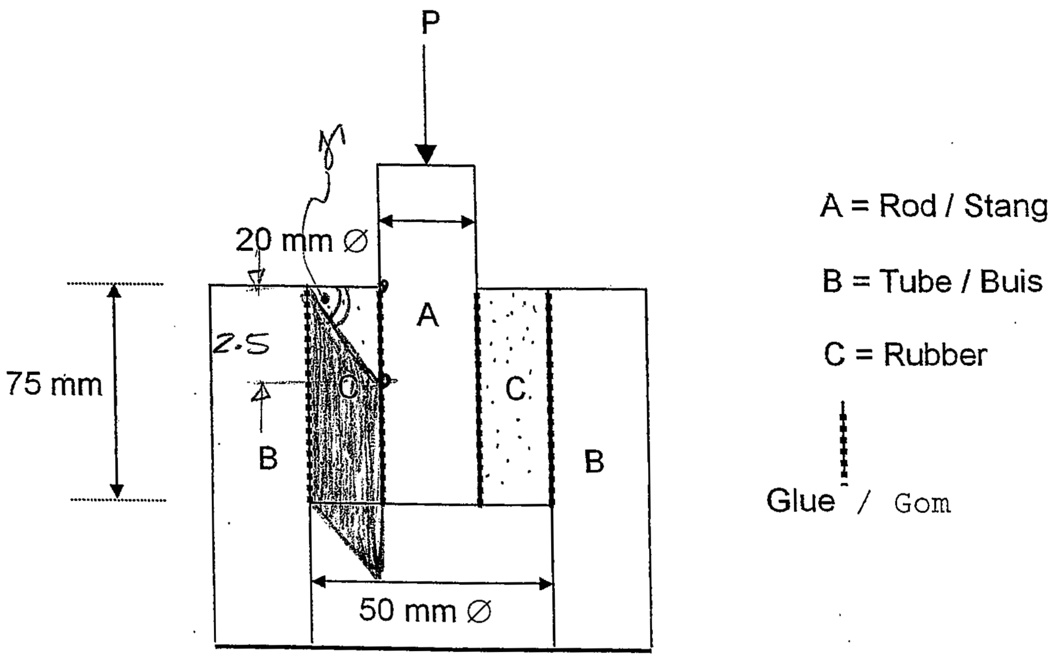
$\Delta d_{in} = 210 * 0.000\ 086 \quad - ②$

$\Delta t = \frac{① - ②}{2} = 1.72\ \mu m \rightarrow$

QUESTION 3 / VRAAG 3 **[8]**

A circular vibration isolator consists of a rod [A] with radius 10 mm and a tube [B] of inner radius 25 mm. The rod and tube are bonded to a 75 mm long hollow rubber cylinder [C] with a Modulus of Rigidity G equal to 12.5 MPa. Determine the maximum value of P provided that the maximum deflection of the rod [A] does not exceed 2.5 mm.

'n Ronde vibrasie isolator bestaan uit 'n stang [A] met radius 10 mm en 'n buis [B] met binneradius 25 mm. Die stang en buis word aanmekaar gebind deur 'n 75 mm lang hol rubbersilinder [C] met Skuifmodulus G gelyk aan 12.5 MPa.
Bepaal die maksimum waarde van P onder voorwaarde dat die maksimum defleksie van die stang [A] nie 2.5 mm oorskry nie.



$\tan \gamma = \frac{2.5}{15} = 0.167$

$\gamma = \tan^{-1}(0.167) = 9.462^\circ = 0.165$ rad

$\tau = G * \gamma$

$\therefore \frac{P}{2\pi(10)(75)} = 12.5\,(0.165)$

$\therefore P = 9719$ N

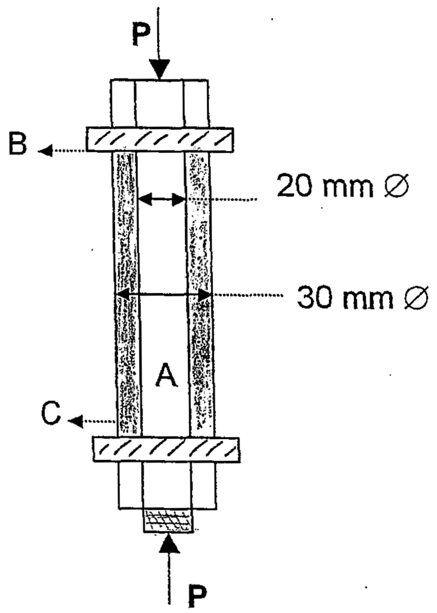
QUESTION 4 / VRAAG 4 [12]

A copper sleeve [C] with outside diameter equal to 30 mm and inside diameter equal to 20 mm surrounds a 20 mm diameter steel bolt [A].
The yield stress for steel is equal to 650 MPa and the yield stress for copper is equal to 500 MPa.

E_{steel} = 200 GPa and E_{copper} = 70 GPa.

'n Koperhuls [C] met buitediameter gelyk aan 30 mm en binnediameter gelyk aan 20 mm omhul 'n 20 mm diameter staal bout. Die vloeispanning van die staal 650 MPa en die vloeispanning van koper gelyk is aan 500 MPa.

E_{staal} = 200 GPa en E_{koper} = 70 GPa.



A = 20 mm ∅ Steel bolt / Staal bout
B = Thin washer / Dun wasser
C = Copper sleeve / Koper huls

4[a] Determine the largest elastic load **P** that can be applied to the assembly if the load P is administered by an external loading. **[8]**
Bepaal die grootste elastiese las **P** wat op die samestelling aangewend kan word indien P toegepas word deur 'n eksterne belasting.

④ $\sigma_s = \sigma_c$

$\therefore \left(\frac{FL}{AE}\right)_s = \left(\frac{FL}{AE}\right)_c$

$$\frac{F_S}{\pi(10)^2 * 200} = \frac{F_C}{\pi(15^2-10^2) * 70}$$

$$\therefore F_S = 2.2857 F_C \quad \ldots (1)$$

$P = F_S + F_C = 3.2857 F_C$	$P = F_S + F_C = 1.4375 F_S$
$= 3.2857\, \sigma_C A_C$	$= 1.4375 * \sigma_S A_S$
$= 3.2857 * 500 * \frac{\pi}{4}(30^2 - 20^2)$	$= 1.4375 * 650 * \frac{\pi}{4} * 20^2$
$= 645 \text{ kN}$	$= 293 \text{ kN}$

$$\therefore P = 293 \text{ kN}$$

4[b] Determine the largest elastic load **P** that can be applied to the assembly if the load P is administered by tightening the bolt. **[4]**

Bepaal die grootste elastiese las **P** wat op die samestelling aangewend kan word indien P toegepas word deur die bout vas te draai.

Steel: $P = \sigma A$

$= 650 * \frac{\pi}{4}(20^2)$

$= 204 \text{ kN}$

Copper: $P = \sigma A$

$= 500 * \frac{\pi}{4}(30^2 - 20^2)$

$= 196 \text{ kN}$

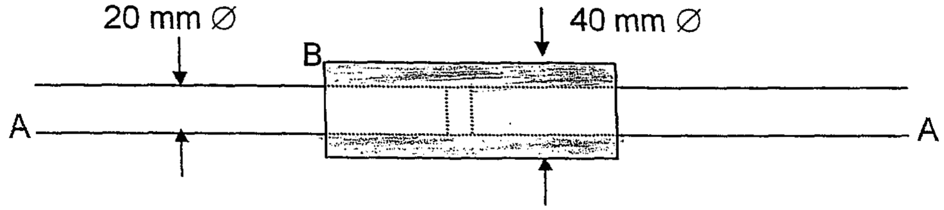
$P = 196 \text{ kN}$

QUESTION 5 / VRAAG 5 [10]

Two steel rods [A] with diameter equal to 20 mm are connected by means of a brass coupling [B].
The brass coupling has an outside diameter equal to 40 mm and fits tightly around the steel rods.
The shear yield stress for steel is 100 MPa.

Twee staalstawe [A] met diameter gelyk aan 20 mm is aanmekaar gekoppel deur middel van brons koppelstuk [B].
Die brons koppelstuk het 'n buitediameter van 40 mm en pas styf oor die staalstawe.
Die skuifvloeispanning vir staal is 100 MPa.

$$\boxed{I_Z = J = \frac{\pi d^4}{32}}$$



5[a] Determine the torque necessary to cause the steel to yield. **[5]**

Bepaal die torsiemoment wat nodig is om die staal te laat vloei.

$$\tau = \frac{Tc}{J}$$

$$100 = \frac{T * 10 * 32}{\pi (20)^4}$$

$$\therefore T = 157\,080 \text{ N.mm}$$

$$= 157 \text{ N.m}$$

5[b] Determine the maximum shear stress in the brass. **[3]**

Bepaal die maksimum skuifspanning in die brons.

$$\tau_{max_B} = \frac{Tc}{J}$$

$$= \frac{157\,079 * 20}{\frac{\pi}{32}(40^4 - 20^4)}$$

$$= 13.33 \text{ MPa} \longrightarrow$$

5[c] Determine the maximum shear stress in the steel. **[2]**

Bepaal die maksimum skuifspanning in die staal.

$$\tau_{max_S} = \frac{Tc}{J}$$

$$= \frac{157\,079 * 10}{\frac{\pi}{32} * 20^4}$$

$$= 100 \text{ MPa} \longrightarrow$$

FORMULA PAGE

Hooke's Law axial stress

$$\sigma = E\varepsilon$$
$$\tau = G\gamma$$
$$G = \frac{E}{2(1+\nu)}$$

Thermal expansion

$$\varepsilon_T = \alpha(\Delta T)$$

Strain Energy (Axial)

$$U = \frac{P^2L}{2EA} = \frac{EA\delta^2}{2L}$$

Strain Energy Density (Axial)

$$u = \frac{\sigma^2}{2E} = \frac{E\varepsilon^2}{2}$$

Impacct Loading

$$\delta_{\max} = \delta_{st}\left[1+\left(1+\frac{2h}{\delta_{st}}\right)^{\frac{1}{2}}\right]$$
$$\delta_{st} = \frac{WL}{EA}$$
$$\sigma_{\max} = \sigma_{st}\left[1+\left(1+\frac{2hE}{L\sigma_{st}}\right)^{\frac{1}{2}}\right]$$
$$\sigma_{st} = \frac{W}{A}$$

Torsion Formulas

$$\tau_{\max} = \frac{Tc}{J};\ \tau = \frac{T\rho}{J}$$

$\theta = \frac{T}{GJ}$ (rate of twist)

$\phi = \frac{TL}{GJ}$ (angle of twist)

Power Transmission

$$P = 2\pi fT \quad (f\ in\ Hz)$$

Strain Energy (Torsional)

$$U = \frac{T^2L}{2GJ} = \frac{GJ\phi^2}{2L}$$

Strain Energy Density (Torsional)

$$u = \frac{\tau^2}{2G} = \frac{G\gamma^2}{2}$$

Flexure Formula

$$\sigma_x = -\frac{My}{I}$$

Section moduli

$S_i = \frac{I}{c_i}$ (i=1,2)

Shear Formula (Rectangular beam)

$$\tau = \frac{VQ}{It}$$

Distribution of shear stress (Rectangular beam)

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$$

Shear Flow

$$q = \frac{VQ}{I}$$

Composite Beams

$$\sigma_{xi} = \frac{MyE_i}{E_1I_1 + E_2I_2}$$

(i=1,2)

Mohr's Circle: Center at

$$\frac{\sigma_x + \sigma_y}{2}$$

Radius

$$R = \left(\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^{\ 2}\right)^{\frac{1}{2}}$$

Hooke's Law for Plane Stress:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$
$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$
$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right)$$

Plane stress energy density:

$$u = \tfrac{1}{2}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z\right)$$

Dilatation:

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Pressure Vessels Spherical:

$$\sigma = \frac{\Pr}{2t}$$
$$\tau_{\max} = \frac{\sigma}{2}$$

Cylindrical:

$$\sigma_1 = \frac{pr}{t}$$
$$\sigma_2 = \frac{pr}{2t}$$
$$\tau_{\max} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

Deflection of Beams

$$EI\frac{d^2v}{dx^2} = M(x)$$