UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA

DEPARTEMENT SIVIELE INGENIEURSWESE
DEPARTMENT OF CIVIL ENGINEERING

STERKTELEER SWK 210 – SEMESTERTOETS 1
STRENGTH OF MATERIALS SWK 210 – SEMESTERTEST 1

8 Maart 2008 / *8 March 2008*

Dosente/*Lecturers*: *Prof L Maree* *Dr E Chaparanganda* *Mnr F van Graan*

Volpunte/*Full Marks* : 60
Tyd/*Time* : 1½ uur/*hrs*

Memorandum

Van en voorletters/*Surname and initials*
Studentenommer/*Student number*
Klastaal / *Class Language*: Afrikaans ☐ English ☐
Studierigting/*Discipline:*

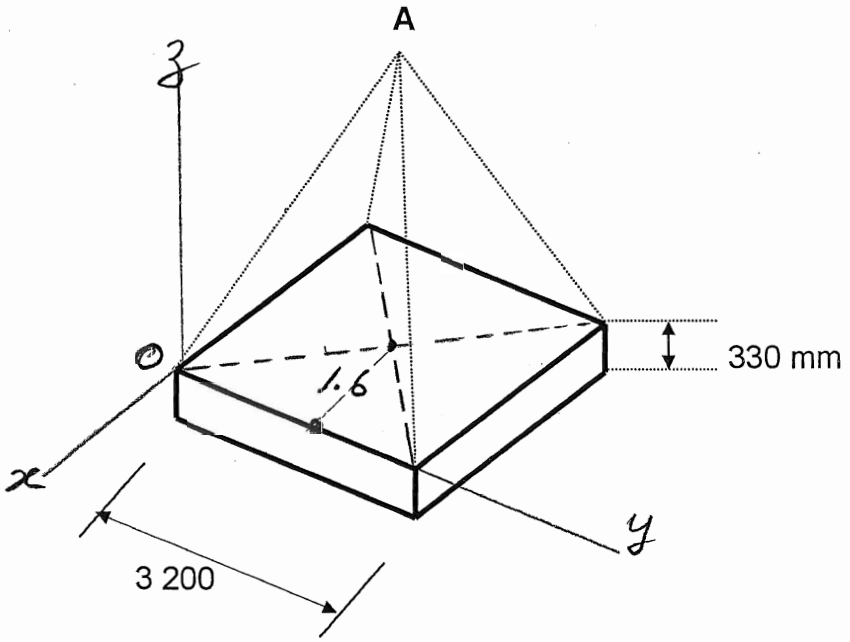
Vraag Question	Punt Mark	
1	8	
2	10	
3	8	
4	12	
5	12	
6	10	
Σ =	**60**	

Vraag Question	Punt Mark	

QUESTION 1 / VRAAG 1 [8]

A prefabricated square concrete slab has a 3.2 meter side length and a thickness of 330 mm.
Four cables of the same length and diameter equal to 15 mm support the slab.
The cables are fixed to the corners of the slab.
The cables are attached to a hook at A.
The vertical distance from the top of the slab to point A is 1.8 metres. A and the centroid of the slab is on the same vertical.
The mass density of concrete is 2 400 kg/m^3 and the yield stress of the steel cables is 505 MPa. Young's Modulus for steel cables = 200 GPa.

'n Voorafvervaardigde vierkantige betonblad het 'n sylengte van 3.2 meter en die bladdikte is 330 mm.
Die blad word ondersteun deur vier kabels wat ewelank is, 'n deursnee van 15 mm het en by die hoeke van die blad vasgeheg is.
Die kabels is geheg aan 'n haak by **A** wat vertikaal bokant die massamiddelpunt van die blad en 1.8 meter hoër as die blad-oppervlakte geleë is.
Die massadigtheid van beton is 2 400 kg/m^3 en die swigspanning van die staalkabels is 505 MPa.
Young se Modulus vir staalkabel = 200 GPa



1[a] Calculate the force in each cable. **[3]**

Bereken die krag in elk van die kabels.

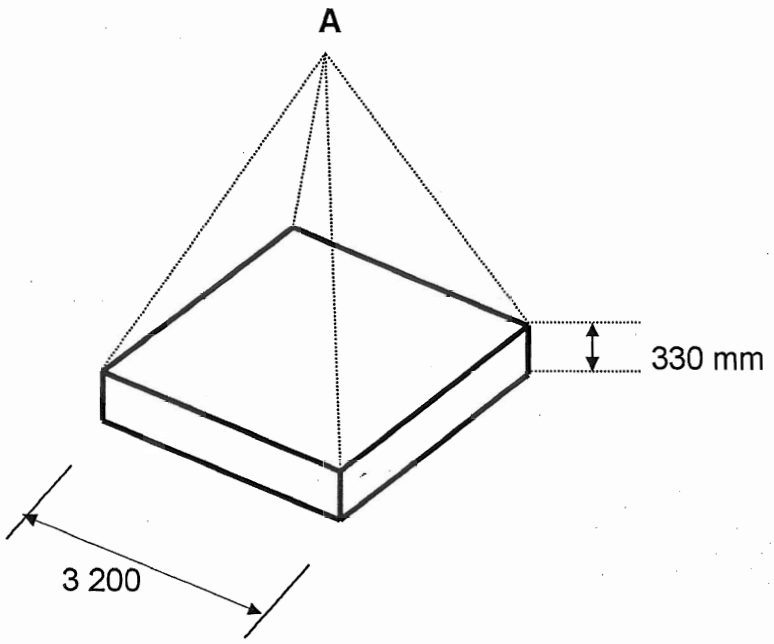
⊙ $\overline{OA} = (A) - (O) = (-1.6; 1.6; 1.8)$ m $OA = 2.891$ m →

⊙ $\overline{F}_{OA} = \frac{F_{OA}}{2.891}(-1.6; 1.6; 1.8)$

⊙ $\sum F_z = 0$:

$\therefore 4 F_{OA}\left(\frac{1.8}{2.891}\right) = 3.2 * 3.2 * 0.33 * 2400 * 9.8$ kN

$\therefore F_{OA} = 3191.3$ N →



1[b] Calculate the change in length in each cable. **[3]**

Bereken die lengteverandering in elk van die kabels.

$$\odot \quad \Delta L = \frac{FL}{AE}$$

$$= \frac{31913 * 2891}{\frac{\pi}{4}(15)^2 * 200 * 10^3}$$

$$= 2{\cdot}610 \text{ mm} \rightarrow$$

1[c] Calculate the factor of safety against a possible failure in the cables. **[2]**

Bereken die veiligheidsfaktor teen 'n moontlike swigting van die kabels.

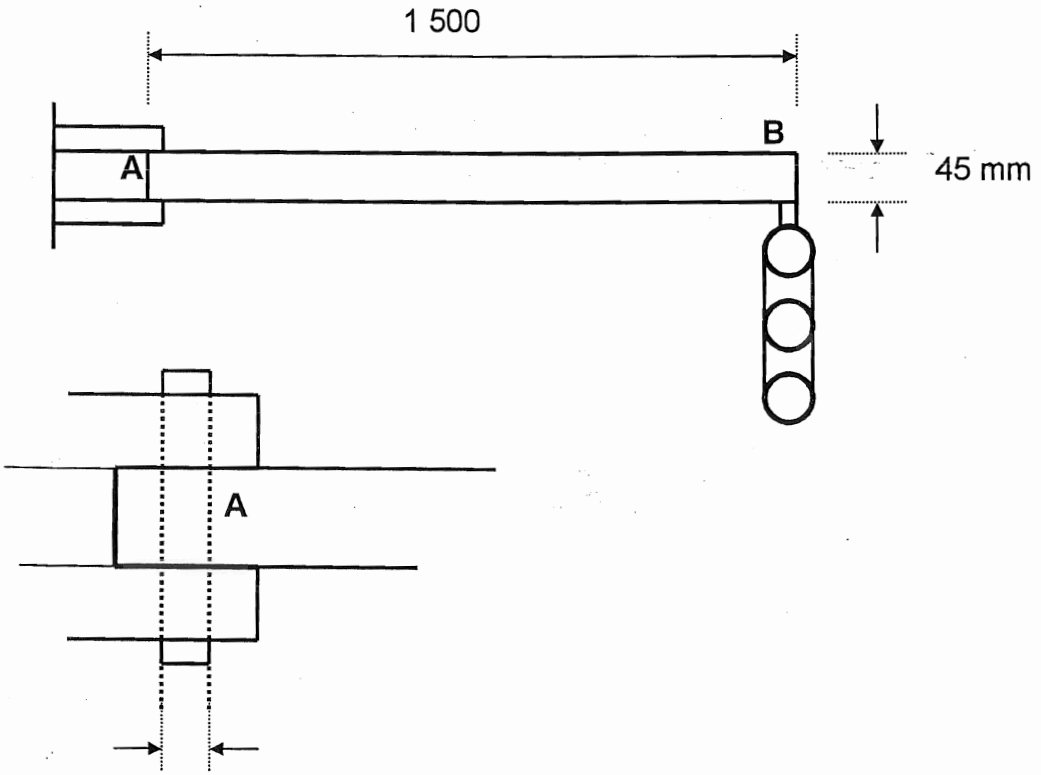
$$\odot \quad \sigma_{OA} = \frac{P}{A} = \frac{31913}{\frac{\pi}{4}(15)^2} = 180{\cdot}59 \text{ MPa}$$

$$\odot \quad FOS = \frac{\sigma_{yield}}{\sigma_{OA}} = \frac{505}{180{\cdot}59} = 2{\cdot}8 \rightarrow$$

QUESTION 2 / VRAAG 2 [10]

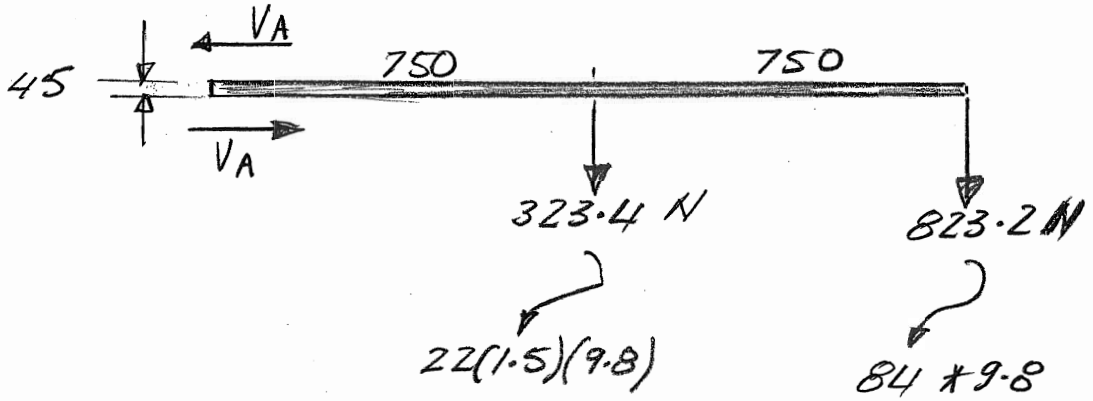
The robot at B has a mass of 84 kg and is supported by a 1.5 metre long robot arm AB, depth 45 mm and a mass of 22 kg / m.
At A a 20 mm diameter pin supports the robot.

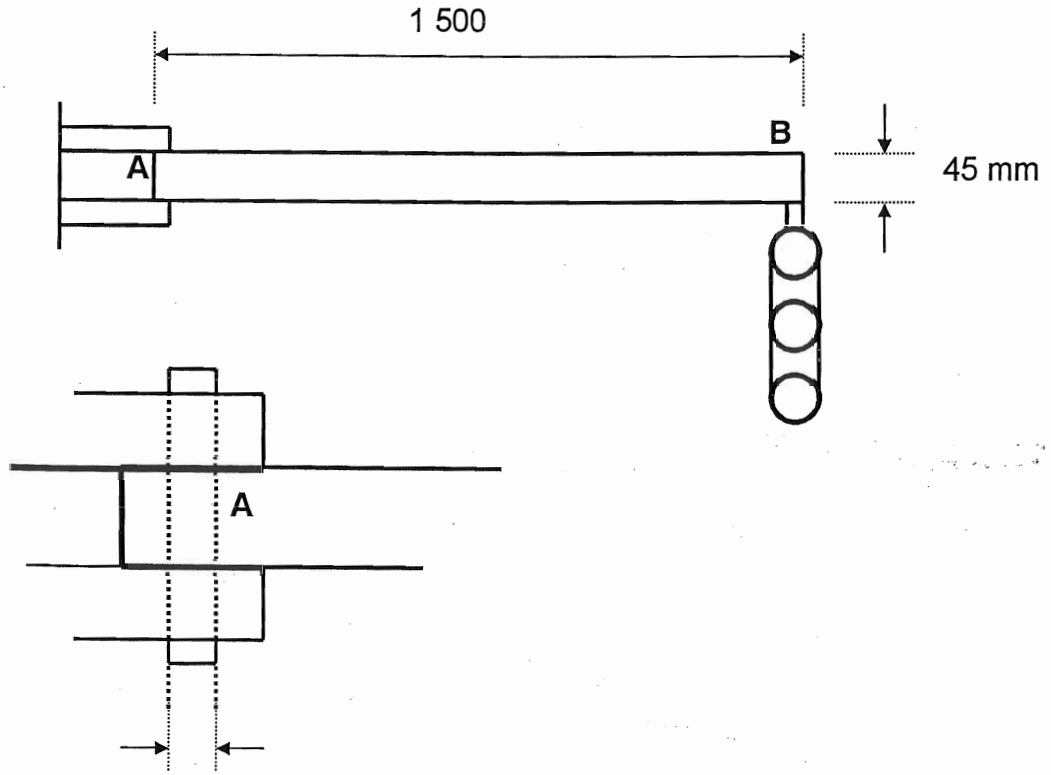
Die verkeerslig het 'n massa van 84 kg en word ondersteun deur 'n 1.5 meter lank arm, diepte 45 mm en 'n massa van 22 kg / m.
Die robot word by A ondersteun deur 'n 20 mm diameter pen.



2[a] Draw the Free Body Diagram for AB. **[2]**

Teken die Vryliggaamskets vir AB.





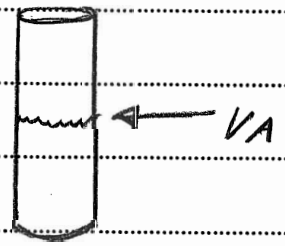
2[b] Calculate the shear stress in the pin at A. Give your anwer in MPa. **[8]**

Bereken die skuifspanning in die pen by A. Gee u antwoord in MPa.

$\circlearrowright \sum M_A = 0$:

$323.4 * 750 + 823.2 * 1500 = 45 V_A$

$\therefore \quad V_A = 32830 \text{ N} \rightarrow$



$\tau = \frac{32830}{\frac{\pi}{4}(20)^2}$

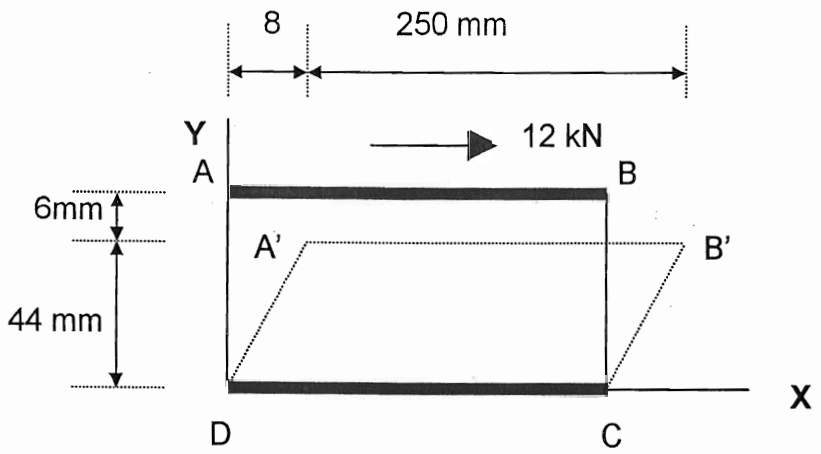
$= 104.5 \text{ N/mm}^2$

$= 104.5 \text{ MPa} \rightarrow$

QUESTION 3 / VRAAG 3 [8]

A nuprene pad is glued between two thin steel plates.
The unit [ABCD] is used as a damper on a bridge support.
The figure shows the damper in operation when an 8 mm lateral deflection at A occurred after application of a 12 kN shear force.
The width of the damper is 150 mm.

'n Nuprene blok is tussen twee dun staalplate vasgegom.
Die eenheid [ABCD] word gebruik as 'n demper by 'n brugondersteuning.
Die figuur toon die demper in werking waartydens die aanwending van 'n 12 kN skuifkrag 'n 8 mm sydelingse verplasing van A veroorsaak het.
Die breedte van die demper is 150 mm.

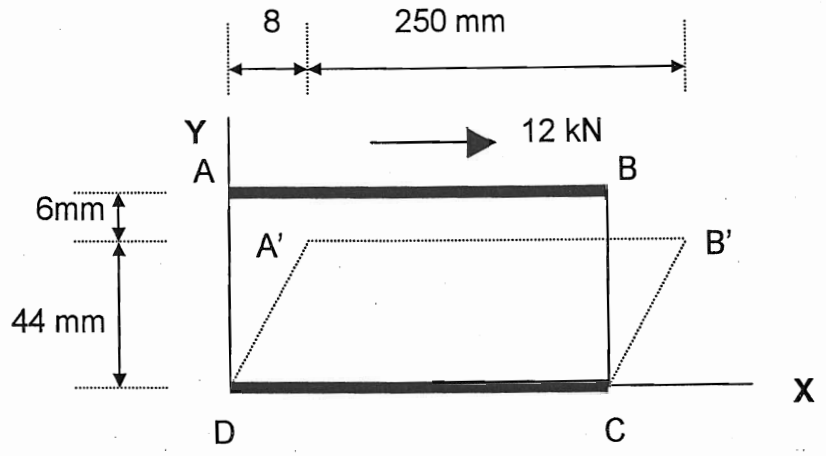


3[a] Calculate the normal lateral strain. Show your calculations [with the correct symbols]. **[2]**

Bereken die normale laterale vervorming. Toon u berekeninge [met die korrekte simbole].

$\varepsilon_x = \frac{0}{250}$

$= 0$



3[b] Calculate the normal vertical strain.
Show your calculations [with the correct symbols]. **[2]**

Bereken die normale vertikale vervorming.
Toon u berekeninge [met die korrekte simbole].

⊙ $\varepsilon_y = \frac{\Delta l}{l} = \frac{-6}{50}$

$= -0.12$ →

3[c] Calculate the shear modules of nuprene.
Show your calculations [with the correct symbols]. **[4]**

Bereken die skuifmodulus van nuprene.
Toon u berekeninge [met die korrekte simbole].

⊙ $\gamma = \tan^{-1}\left(\frac{8}{44}\right) = 10.31^\circ$

$= 0.1799$ rad →

⊙ $\tau = \frac{V}{A} = \frac{12000}{250 * 150} = 0.32$ MPa →

⊙ $G = \frac{\tau}{\gamma} = \frac{0.32}{0.1799} = 1.779$ MPa →

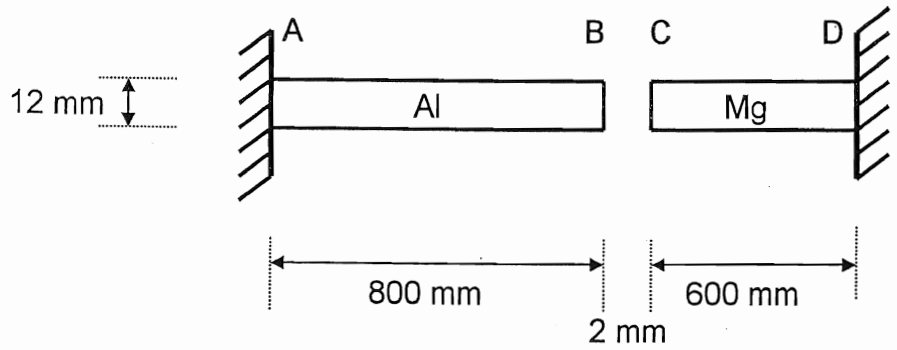
QUESTION 4 / VRAAG 4 [12]

An aluminium bar AB, 800 x 20 x 12 mm, is horizontally fixed at A and an magnesium bar CD, 600 x 20 x 12 mm, is horizontally fixed at D.
The gap at BC between the two bars is 2 mm and the temperature is 25°C.

E_{al} = 70 GPa and α_{al} = 22 x 10^{-6} / °C. **E_{mg} = 45 GPa and α_{mg} = 30 x 10^{-6} / °C.**

'n Aluminium staaf AB, 800 x 20 x 12 mm, is horisontaal geheg by A en 'n magnesium staaf CD, 600 x 20 x 12 mm, is horisontaal geheg by D.
Die gaping by BC tussen die twee stawe is 2 mm en die temperatuur is 25°C.

E_{al} = 70 GPa en α_{al} = 22 x 10^{-6} /°C. **E_{mg} = 45 GPa en α_{mg} = 30 x 10^{-6} / °C.**



4[a] Calculate the temperature at which the 2 mm gap will just close. **[4]**

Bereken die temperatuur waarby die 2 mm gaping net sal sluit.

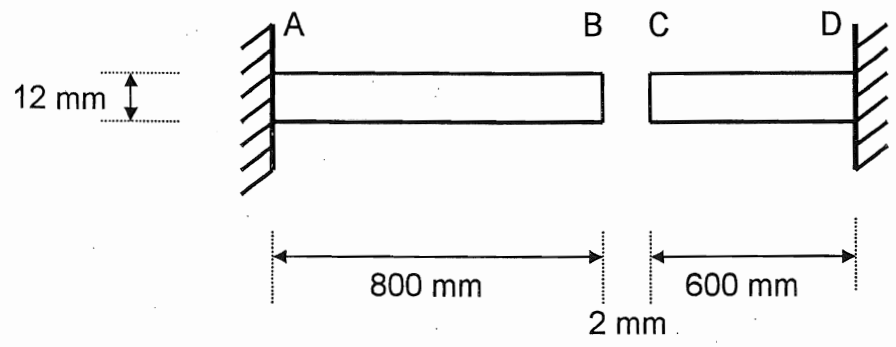
$\delta = L \alpha \Delta T$

$2 = 800 * 22 * 10^{-6} * \Delta T + 600 * 30 * 10^{-6} * \Delta T$

$\therefore 35.6 * 10^{-3} \Delta T = 2$

$\Delta T = 56.2°$ →

$\therefore T2 = 56.2 + 25 = 81.2°C$ →



E_{al} = 70 GPa and / en α_{al} = 22 x 10^{-6} / °C. E_{mg} = 45 GPa and / en α_{mg} = 30 x 10^{-6} / °C.

4[b] Assume that the temperature rises to 110°C.
Calculate the axial force in each plate. **[8]**

Veronderstel dat die temperatuur styg tot 110 °C.
Bereken die aksiale krag in elke plaat.

$$\textcircled{a}(\alpha \Delta T L)_{Al} + (\alpha \Delta T L)_{mg} = 2 + \Delta L_{F_{al}} + \Delta L_{F_{mg}}$$

$$LHS = 22*10^{-6}*85*800 + 30*10^{-6}*85*600$$

$$= 3.026\ mm \longrightarrow \quad \cdots (1)$$

$$RHS = 2 + \left(\frac{F*L}{AE}\right)_{Al} + \left(\frac{FL}{AE}\right)_{mg} \qquad (F_{Al} = F_{mg})$$

$$= 2 + \frac{F*800}{20(12)(70)10^3} + \frac{F*600}{20(12)(45)(10^3)} \quad \cdots (2) \longrightarrow$$

$$(1) = (2): \quad 1.026 = F*(10.3175)*10^{-5}$$

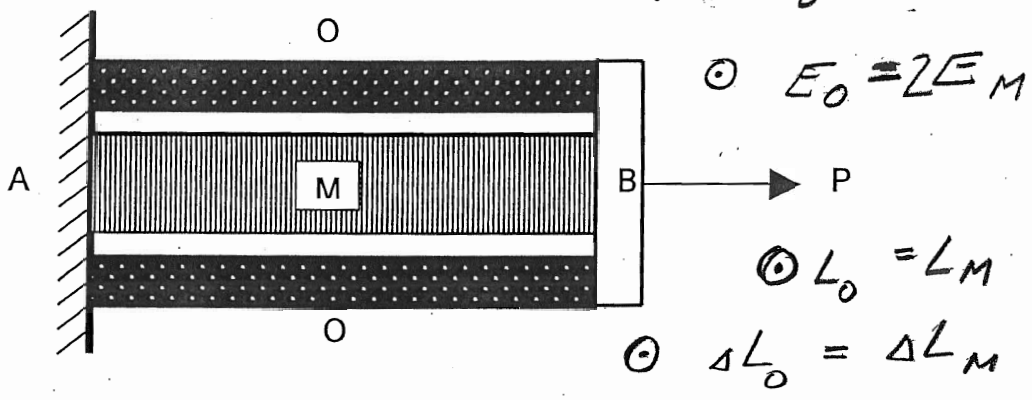
$$\therefore F = 9944.3\ N \longrightarrow$$

QUESTION 5 / VRAAG 5 [12]

The three bars are fixed at A and rigidly bonded at B.
The two outer bars are identical, made of material O and the middle bar is made of material M.
An outer bar has a cross section of 20 mm x 20 mm and the middle bar has a cross section of 20 mm x 30 mm.
The modulus of elasticity of an outer bar is twice that of the middle bar.
A load **P** is applied to the set-up.

Die drie stawe is ingebou by A en star verbind by B.
Die twee buitenste stawe is identies, gemaak van materiaal O en die middelste staaf is gemaak vam material M.
Die buitenste stawe het 'n dwarsdeursnit van 20 mm x 20 mm en die middelste staaf het 'n 'n dwarsdeursnit van 20 mm x 30 mm.
Die modulus van elastisisteit van 'n buitestaaf is dubbel die waarde van die middelste staaf.
'n Las **P** word op die samestelling aangewend.

$1.5\,A_O = A_M$



$\odot\ E_O = 2E_M$

$\odot\ L_O = L_M$

$\odot\ \Delta L_O = \Delta L_M$

5[a] What percentage of the load P is transmitted by the middle bar (bar M)? **[8]**

Watter persentasie van die las P word deur die middelste staaf (staaf M) oorgedra?

$\odot\ 2F_O + F_M = P \quad \ldots\ (1)$

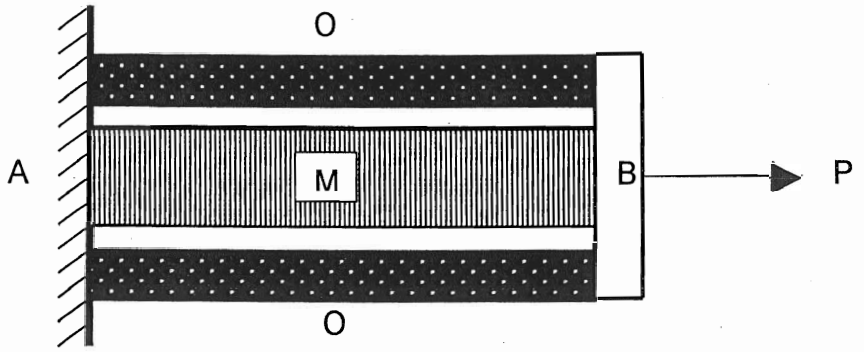
$\odot\ \Delta L_O = \Delta L_M \quad \therefore \left(\frac{FL}{AE}\right)_O = \left(\frac{FL}{AE}\right)_M$

$\therefore \frac{F_O \times L}{\frac{A_M}{1.5} \times 2E_M} = \frac{F_M \times L}{A_M \times E_M}$

$\therefore \frac{1.5F_O}{2} = F_M \quad \therefore F_M = 0.75F_O \quad \ldots(2)$

$(1);(2): \ 2\left(\frac{F_M}{0.75}\right) + F_M = P \quad \therefore F_M = 0.272P$

$\therefore F_M = 27.2\%$ of P →



5[b] What is the ratio of the stress in the middle bar to the stress in the outer bars? **[2]**

Wat is die verhouding van die spanning in die middelste staaf tot die spanning in die buitenste staaf?

$$\frac{\sigma_M}{\sigma_O} = \frac{\varepsilon_M * E_M}{\varepsilon_O * E_O} = \frac{E_M}{E_O} \quad (\varepsilon_O = \varepsilon_M)$$

$$= \frac{E_M}{2E_M} = \frac{1}{2} \rightarrow$$

$$\therefore \sigma_M = \frac{\sigma_O}{2} \rightarrow$$

5[c] What is the ratio of the strain in the middle bar to the strain in the outer bars? **[2]**

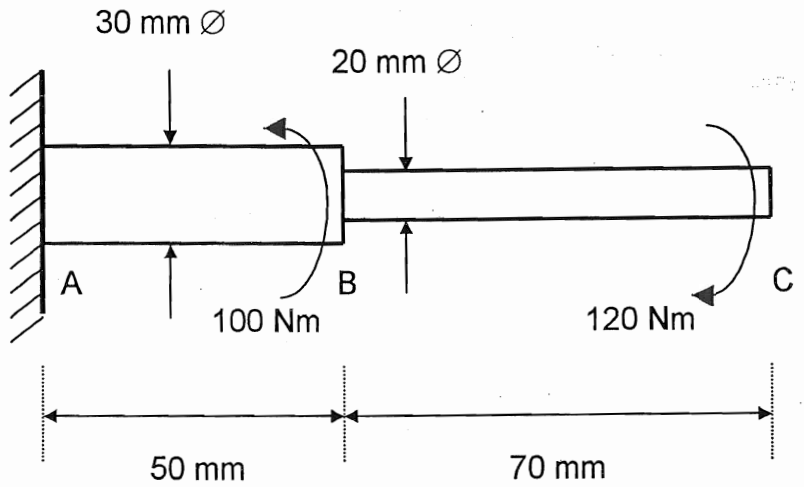
Wat is die verhouding van die vervorming in die middelste staaf tot die vervorming in die buitenste staaf?

$$\frac{\varepsilon_M}{\varepsilon_O} = \frac{\left(\frac{\Delta L}{L}\right)_M}{\left(\frac{\Delta L}{L}\right)_O} = 1$$

QUESTION 6 / VRAAG 6 [10]

Part AB of the solid circular metal shaft ABC is 50 mm long and its diameter is 30 mm.
Part BC is 70 mm long and its diameter is 20 mm.
Torques of 100 Nm and 120 Nm respectively are applied at B and C. G_{metal} = 70 GPa

Deel AB van die soliede ronde metaalstaaf ABC is 50 mm lank en die diameter is 30 mm.
Deel BC is 70 mm lank en die diameter is 20 mm.
Torsiemomente van 100 Nm en 120 Nm respektiewelik word by B en C toegepas. G_{metaal} = 70 GPa



$$I_Z = \frac{\pi}{32} d^4$$

6[a] Calculate the maximum shear stress in the bar. [7]

Bereken die maksimum skuifspanning in die staaf.

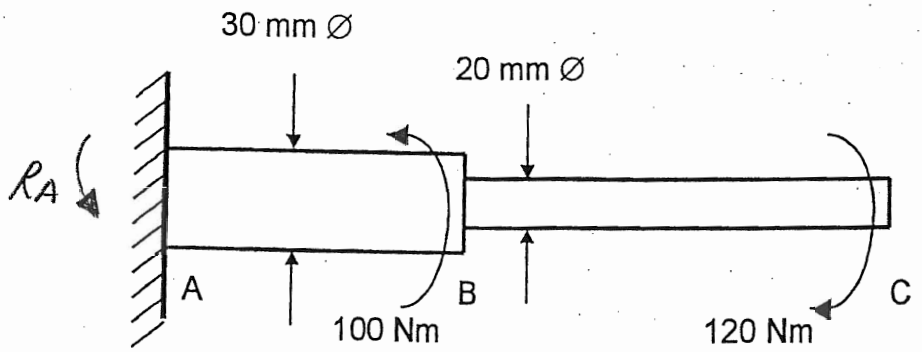
⊙

BC: $\tau_{max} = \frac{T\rho}{I_p} = \frac{120 * 10^3 * 10}{\pi/32 * 20^4} = 76.4$ MPa →

AB: $\tau_{max} = \frac{Tr}{I_p} = \frac{(120-100) * 10^3 * 15}{\pi/32 * 30^4} =$

$= 3.77$ MPa →

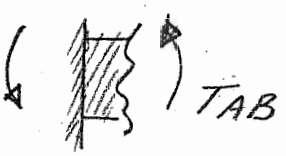
$\therefore \tau_{max} = 76.4$ MPa →



⊙ Reaction at A = R_A

$\Sigma T = 0 \quad \therefore \quad R_A + 100 - 120 = 0 \quad \therefore R_A = 20\ N.m$

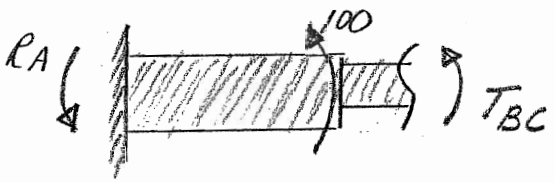
For AB:



$\Sigma T = 0 \quad \therefore R_A + T_{AB} = 0$

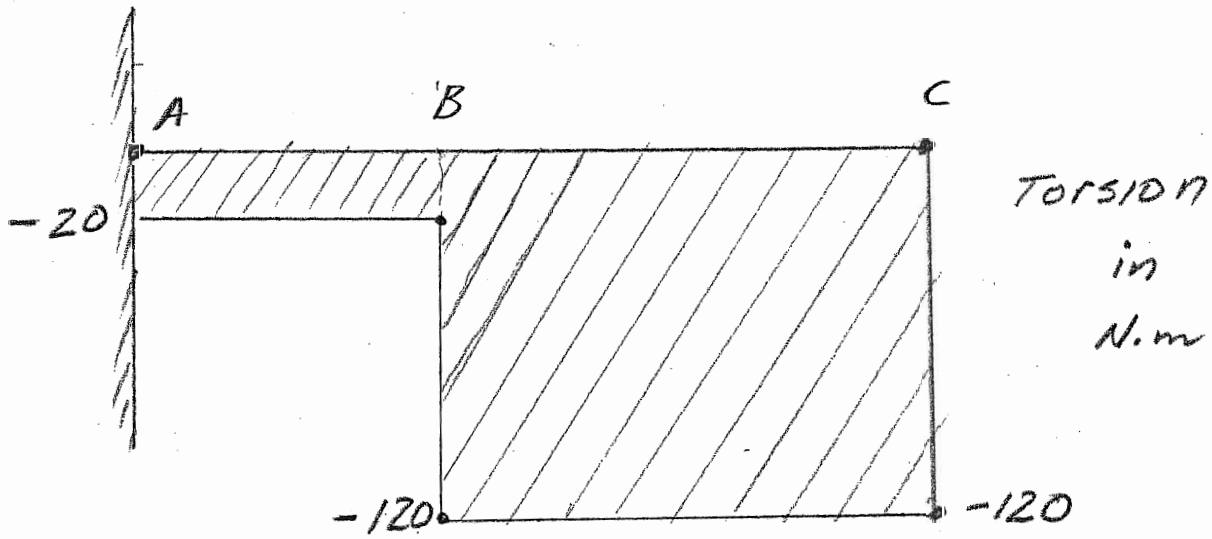
$\therefore 20 + T_{AB} = 0 \quad \therefore T_{AB} = -20\ N.m$

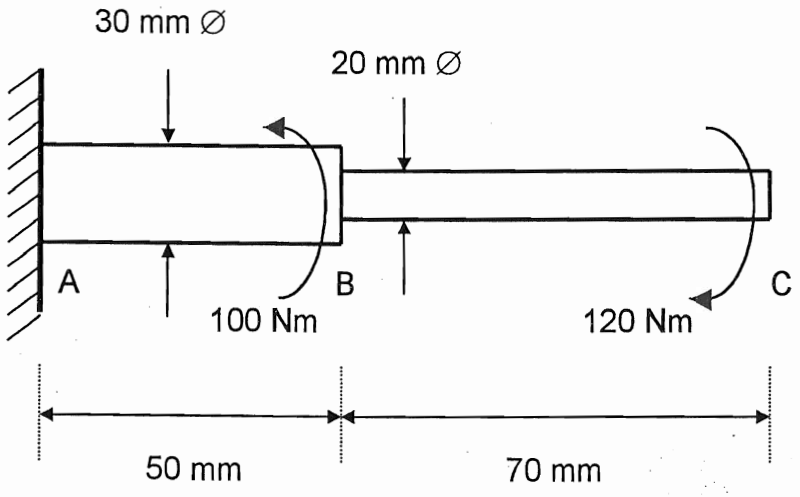
For BC:



$\Sigma T = 0 \quad R_A + 100 + T_{BC} = 0$

$\therefore 20 + 100\ T_{BC} = 0 \quad \therefore T_{BC} = -120\ N.m$





6[b] Determine the angle of twist at point C of the bar.
Give your answer in degrees. **[3]**

Bepaal die wringingshoek by punt C van die staaf.
Gee u antwoord in grade.

$$\phi = \frac{TL}{GJ}$$

$$\phi_C = \phi_{AB} + \phi_{BC}$$

$$= \frac{20 * 10^3 * 50}{70 * 10^3 * \frac{\pi}{32}(30)^4} + \frac{120 * 10^3 * 70}{70 * 10^3 * \frac{\pi}{32}(20)^4}$$

$$= 0.0001796\,5 + 0.007394$$

$$= 0.007\,819 \text{ rad}$$

$$= 0.448^\circ \longrightarrow$$